# Inequality 16

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Q> $a, b, c$ are sides of $\Delta ABC$ with area $\Delta$. Prove that $\underline{ab+bc+ca \geq 4\sqrt{3}\Delta}$ and equality holds iff $\Delta ABC$ is equilateral.

Ans:- $x = \frac{a+b+c}{2} - a = \frac{b+c-a}{2}$ $\quad y = \frac{c+a-b}{2}$ $\quad z = \frac{a+b-c}{2}$

$x+y = c \quad y+z = a \quad z+x = b$

This is the inequality to prove

$$(x+y)(y+z) + (y+z)(z+x) + (z+x)(x+y) \geq 4\sqrt{3}\sqrt{(x+y+z)xyz}$$

$$LHS = xy + xz + y^2 + yz + yz + yx + z^2 + zx + zx + zy + x^2 + xy$$

$$= x^2+y^2+z^2+3(xy+yz+zx)$$

because $x^2+y^2+z^2 \geq xy+zx+yz$

$$\geq \frac{1}{3}(x^2+y^2+z^2) + \frac{2}{3}(xy+yz+zx) + 3(xy+yz+zx)$$

$$= \frac{1}{3}(x+y+z)^2 + 3(xy+yz+zx) \underset{AM-GM}{\geq} 4\sqrt[4]{\frac{1}{\cancel{3}}(x+y+z)^2 x^2y^2z^2 3^{\cancel{3}2}}$$

$$= 4\sqrt{3(x+y+z)xyz}$$

Q> $z_1 = \sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}$ $\quad z_2 = \sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$ $\quad z_3 = \sqrt[6]{\frac{1+i}{\sqrt{3}-i}}$

a> $\sum |z_i|^2 = \frac{3}{2}$ $\qquad$ b> $|z_1|^4 + |z_2|^4 = |z_3|^{-8}$

c> $\sum |z_i|^3 + |z_2|^3 = |z_3|^{-6}$ $\qquad$ d> $|z_1|^4 + |z_2 = |z_3|^8$

Ans:- $z_1 = \sqrt[6]{\frac{1-i}{1+i\sqrt{3}}} = \sqrt[6]{\frac{\sqrt{3}-1}{4} + i\frac{\sqrt{3}+1}{4}} = \frac{1}{\sqrt[6]{4}}\sqrt[6]{(2\sqrt{2})\frac{1}{2\sqrt{2}}(\sqrt{3}-1+i\sqrt{3}+1)}$

$|e^{i\theta}| = 1$

$\Rightarrow p^2\left((\sqrt{3}-1)^2 + (\sqrt{3}+1)^2\right) = 1$

$\Rightarrow p^2(3+1-\cancel{2\sqrt{3}}+3+1+\cancel{2\sqrt{3}}) = 1$

$\Rightarrow p = \frac{1}{2\sqrt{2}}$

$$= \frac{\sqrt[6]{2\sqrt{2}}}{\sqrt[6]{4}} e^{i\theta/6}$$

$$|z_1| = \frac{(\sqrt{2})^{3/6}}{\sqrt{2}^{4/6}} = 2^{\frac{3}{12}-\frac{4}{12}} = 2^{-1/12} = \left(\frac{1}{2}\right)^{1/12}$$

Q> If $z_1, z_2, z_3$ are three distinct complex no such that $\frac{3}{\ldots} = \frac{4}{\ldots} = \frac{5}{|z_1 - z_2|}$ then the value of

Q5/ If $z_1, z_2, z_3$ are three distinct complex no such that

$$\frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|}$$ then the value of

$$\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2}$$

$z\bar{z} = |z|^2$

Ans:- $$\frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|} = k$$

$$= k^2 (0) = 0$$

$$\Rightarrow 9 = k^2 (z_2 - z_3)(\bar{z}_2 - \bar{z}_3)$$

$$16 = k^2 (z_3 - z_1)(\bar{z}_3 - \bar{z}_1)$$

$$25 = k^2 (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$